

# Probing Multivariate Indicators for Academic Evaluation

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**Abstract:** *We combine the Integrated Impact Indicator (I3) and the h-index into the I3-type publication score  $I3X=X_1+X_2+X_3$  and the citation score  $I3Y=Y_1+Y_2+Y_3$ . The publication vector  $X_1+X_2+X_3$  and the citation vector  $Y_1+Y_2+Y_3$  are based on percentile classes generated by the h-index. These multivariate indicators can be used for academic evaluation. The empirical studies show that the h-core distribution is suitable to evaluate scholars, the  $X_1$  and  $Y_1$  are applied to measure core impact power of universities, and  $I3X$  and  $I3Y$  are alternatives of journal impact factor (JIF). The multivariate indicators provide a multidimensional view of academic evaluation using the advantages of both the h-index and I3: (i) the publications and not only the citations are appreciated; (ii) the indicators are non-parametric; (iii) the results are easy to obtain from WoS or Scopus data; (iv) the results can be plotted (XY).*

**Keywords:** *I3; h-index; publications; citations; multivariate indicators; academic evaluation*

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## 1. Introduction

Academic evaluation has continued to be an issue in the academic world, as it is difficult to select and set universal evaluating principles in various complicated situations. However, publications and citations remain the main focus of academic evaluation, particularly for fundamental research. Citations cannot directly be compared with publications and thus one needs a model or at least a formula. A model can be improved and thus the measurement be refined. Since all models also generate error, the quality of a model depends on the quality of the arguments used for constructing the model. What the advantages and disadvantages?

Since Garfield introduced the journal impact factor (JIF) and set up citation analysis (Garfield, 1955, 1979), these scientometric indicators have been applied into academic evaluation. Hirsch (2005) proposed the h-index, which was quickly accepted by the academic world. This promoted the research and development of quantitative academic indicators.

Both JIF and h-index had their advantages and disadvantages, with basic designed differences concerning JIF for journals and h-index for scholars respectively. After developing a set of criteria for an indicator in Leydesdorff et al. (2011), these authors proposed the Integrated Impact Indicator *I3* (Leydesdorff & Bornmann 2011). *I3* is based on (i) transformation of the citation distribution into a distribution of quantiles and (ii) integration (instead of averaging) of the quantile values. (Quantiles are the continuous equivalent of percentiles.) The use of percentiles was recently recommended in the Leiden Manifesto (“Ten principles to guide research evaluation”; Hicks et al., 2015), because average citation rates are heavily dependent on the few highly cited papers in a publication set and the distributions are very skewed. *I3* combines citation impact and publication output into a single number – similar to the *h*-index.

The quantile values which are conveniently normalized between zero and hundred provide the weights for the papers, as follows:

$$I3(i) = \sum_{i=1}^C f(X_i) \cdot X_i \quad (1)$$

where  $X_i$  indicates the percentile ranks and  $f(X_i)$  denotes the frequencies of the ranks with  $i=[1,C]$  as the percentile rank classes, which means that the measures  $X_i$  are divided into  $C$  classes each with a scoring function  $f(X_i)$  or weight ( $w_i$ ), so that one can also re-write Eq. (1) as follows:

$$I3(i) = \sum_i w_i X_i; \sum_i w_i = 1 \quad (2)$$

As an alternative to quantiles, the  $h$  value of a document set can be used to provide a rank class structure. This combines the advantages of  $I3$  and  $h$  into a single framework (Rousseau & Ye, 2012; Ye & Leydesdorff, 2014), which can be applied to academic evaluation based on publications and citations at both group and individual levels. In this study, we extend the methodology which was previously applied to journals (Ye et al., 2017) to universities as well as individual scholars.

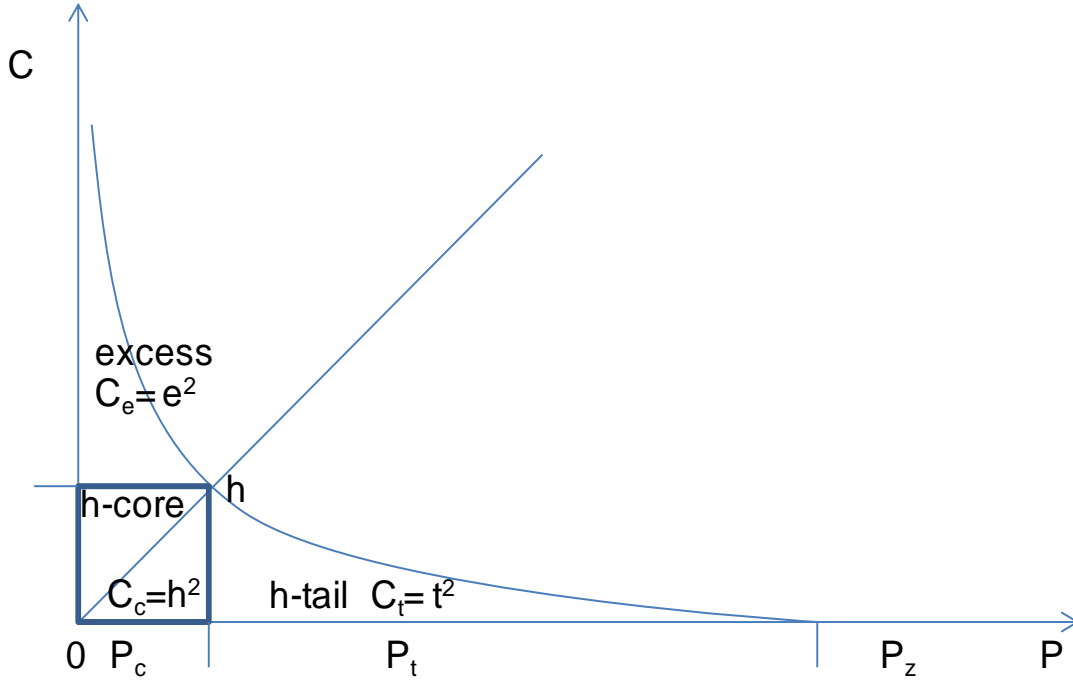
## 2. Methodology

In many cases, single numbers are used as indicators in academic evaluations. However, a single number can only reflect one side of the overall information and can therefore be expected

to have limitations and disadvantages. Possible solutions are multivariate indicators which reflect the multidimensional information. The h-based I3-type multivariate indicators provided a framework of an elaborate methodology (Ye et al., 2017).

### 3.1 Methods

Let us assume that the y-axis denotes citations and the x-axis indicates ranked publications from high citation to low citation, then we obtain a publication-citation distribution as in Figure 1. The  $h$ -index allows us to define three rank classes of both publications and citations in Figure 1. The three classes of publications along the x-axis are: (i) publications in the  $h$ -core (Ye & Rousseau, 2010; Chen *et al.*, 2013)  $P_c$ , (ii) publications in the  $h$ -tail  $P_t$ , (iii) and publications without citations  $P_z$ . Along the y-axis of the citations one can analogously distinguish among (i) the “excess citations” in the  $h$ -core (Zhang, 2009, 2013)  $C_e=e^2$ , (ii) citations to publications in the  $h$  square of the  $h$ -core  $C_c=h^2$ , and (iii) citations to publications in the  $h$ -tail  $C_t=t^2$ .



**Fig. 1** The rank distribution of citations versus publications.

Let  $x_c=P_c/(P_c+P_t+P_z)$ ,  $x_t=P_t/(P_c+P_t+P_z)$ ,  $x_z=P_z/(P_c+P_t+P_z)$ ,  $y_c=C_c/(C_c+C_t+C_e)$ ,  $y_t=C_t/(C_c+C_t+C_e)$  and  $y_e=C_e/(C_c+C_t+C_e)$ , we may define two independent vectors:

$$\mathbf{X} = (X_1, X_2, X_3) = (x_c P_c, x_t P_t, x_z P_z) = (P_c^2 / P, P_t^2 / P, P_z^2 / P) \quad (3)$$

$$\mathbf{Y} = (Y_1, Y_2, Y_3) = (y_c C_c, y_t C_t, y_e C_e) = (C_c^2 / C, C_t^2 / C, C_e^2 / C) \quad (4)$$

as well as an *I3*-type publication indicator *I3X* and an *I3*-type citation indicator *I3Y* as follows

$$I3X = x_c P_c + x_t P_t + x_z P_z = X_1 + X_2 + X_3 \quad (5)$$

$$I3Y = y_c C_c + y_t C_t + y_e C_e = Y_1 + Y_2 + Y_3 \quad (6)$$

The vector  $\mathbf{X}$  and the score  $I3X$  represent the relative frequencies of the publications, while the vector  $\mathbf{Y}$  and the score  $I3Y$  denote the relative frequencies of the citations. For convenient application, citation score in h-core can be merged into  $Y_h = Y_1 + Y_3 = y_h C_h$ , where  $y_h = C_h / C$ ,  $C_h = C_e + C_c$ .

Thus, the h-based I3-type multivariate indicators provide multidimensional indicators:  $X_1$  measures publication score in the h-core ( $X_1$  and  $Y_1$  combination may measure core impact power),  $X_2$  measures publication score in h-tail,  $Y_h$  measures citation score in h-core,  $Y_2$  measures citation score in h-tail,  $I3X$  does total publication score, and  $I3Y$  does total citation score.

### 3.2 Data

Since  $P = P_c + P_t + P_z$ ,  $C = C_h + C_t = C_c + C_t + C_e$ ,  $C_h = C_c + C_e$ ,  $P_c = h$ ,  $C_c = h^2$ , one needs to measure only five independent numbers,  $P$ ,  $C$ ,  $P_z$ ,  $C_h$ ,  $h$ , for the computation of  $X$  and  $Y$ ,  $I3X$  and  $I3Y$ , via  $P_t = P - P_c - P_z$ ,  $C_c = h^2$ ,  $C_t = C - C_h$ , and  $C_e = C_h - C_c$ . These five values can be obtained easily from bibliometric databases, like by searching Web of Science (WoS) or Scopus..

In order to show the general applicability of these measures, we provide three examples at different levels: 1) individual scholars, we choose the profiles of ourselves in order to avoid issues concerning personal records and privacy using 10 years of data from WoS 2005-2015; 2) universities: we chose 25 famous universities, including nine in the USA, nine in China, two in the UK and Germany respectively, and single ones from Australia, Canada, and Japan, with five year data from 2011 to 2015 in WoS; 3) journals, we chose journal datasets 2011- 2015, in the field of electrochemistry(EC). The parameters computed from the datasets are listed in the

appendix. We also collected 2009-2013 data of 25 famous universities and the journal data 2011-2015 in the field of history of the social sciences (HSS), for comparative applications.

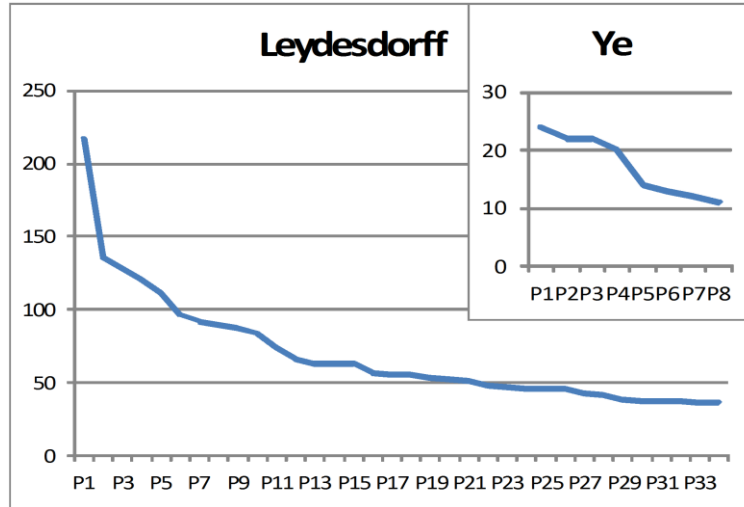
### 3. Results

The publication vector  $\mathbf{X} = (X_1, X_2, X_3)$  and the citation vector  $\mathbf{Y} = (Y_1, Y_2, Y_3)$  are represented by distributed numbers, which are listed in the appendix. The distributed numbers reflect multidimensional academic information, so that the multivariate vectors  $\mathbf{X}$  and  $\mathbf{Y}$  contribute possible applications as multidimensional indicators. If we want to compare research objects to one another, we can inspect the tabled values of publication vector  $\mathbf{X}$  and citation vector  $\mathbf{Y}$ , where  $(X_1, X_2, X_3)$  and/or  $(Y_1, Y_2, Y_3)$  rank accordingly. However, if we merge the same-type numbers into one indicator, *I3*-type indicators can be a good choice.  $I3X = X_1 + X_2 + X_3$  and  $I3Y = Y_1 + Y_2 + Y_3$  sum the scores of vector  $\mathbf{X}$  and  $\mathbf{Y}$ , respectively. All scores can be plotted into figures.

#### 3.1 Individual level: scholars

The scholars' data can be searched on the basis of definite field and time span in definite database. Individual dataset is small, so that all indicators can be easily calculated, such as h-index,  $X_i$ ,  $Y_i$ ,  $I3X$ ,  $I3Y$ , even h-core and h-tail distributions of publications and citations.

Figure 2 shows the h-core distributions of Leydesdorff L and Ye FY.



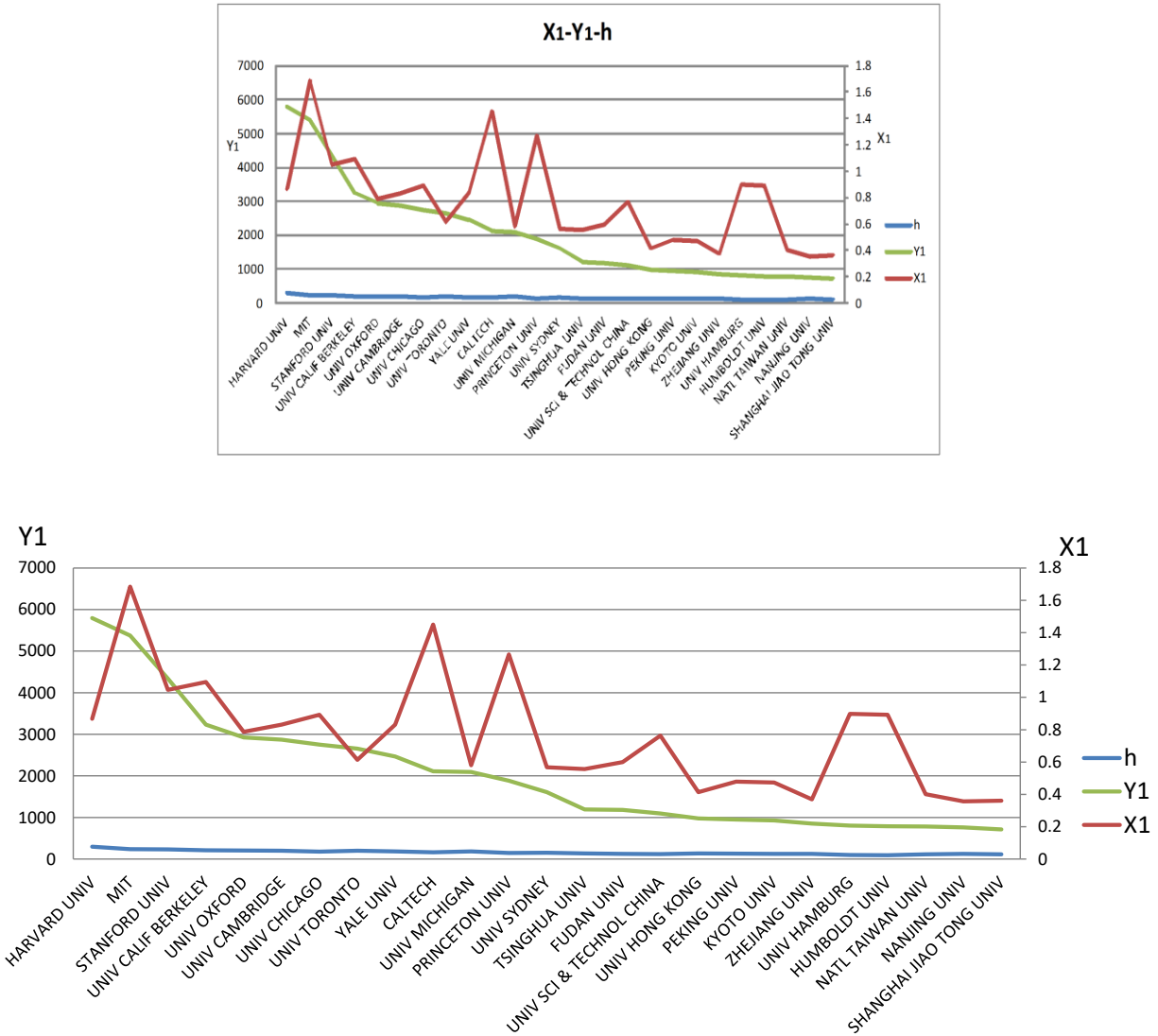
**Fig. 2** Leydesdorff's and Ye's h-core

If the representatives of a scholar come from his/her publications in his/her h-core, the multivariate indicators supplies a feasible way for mining the representatives. Meanwhile, for younger scholars with small h-index, the indicators  $X_2$  and  $Y_2$  can be used to indicate their potential.

### 3.2 Group level: universities

For any university, there are lots of publications and citations distributed in many fields, so that the multivariate indicators provide useful indicators from different perspectives. As we concern the core impact power, the h-index,  $X_1$  and  $Y_1$  may provide important h-core information, while ignoring the h-tail. Figure 3 shows the core impact power of 25 famous universities.



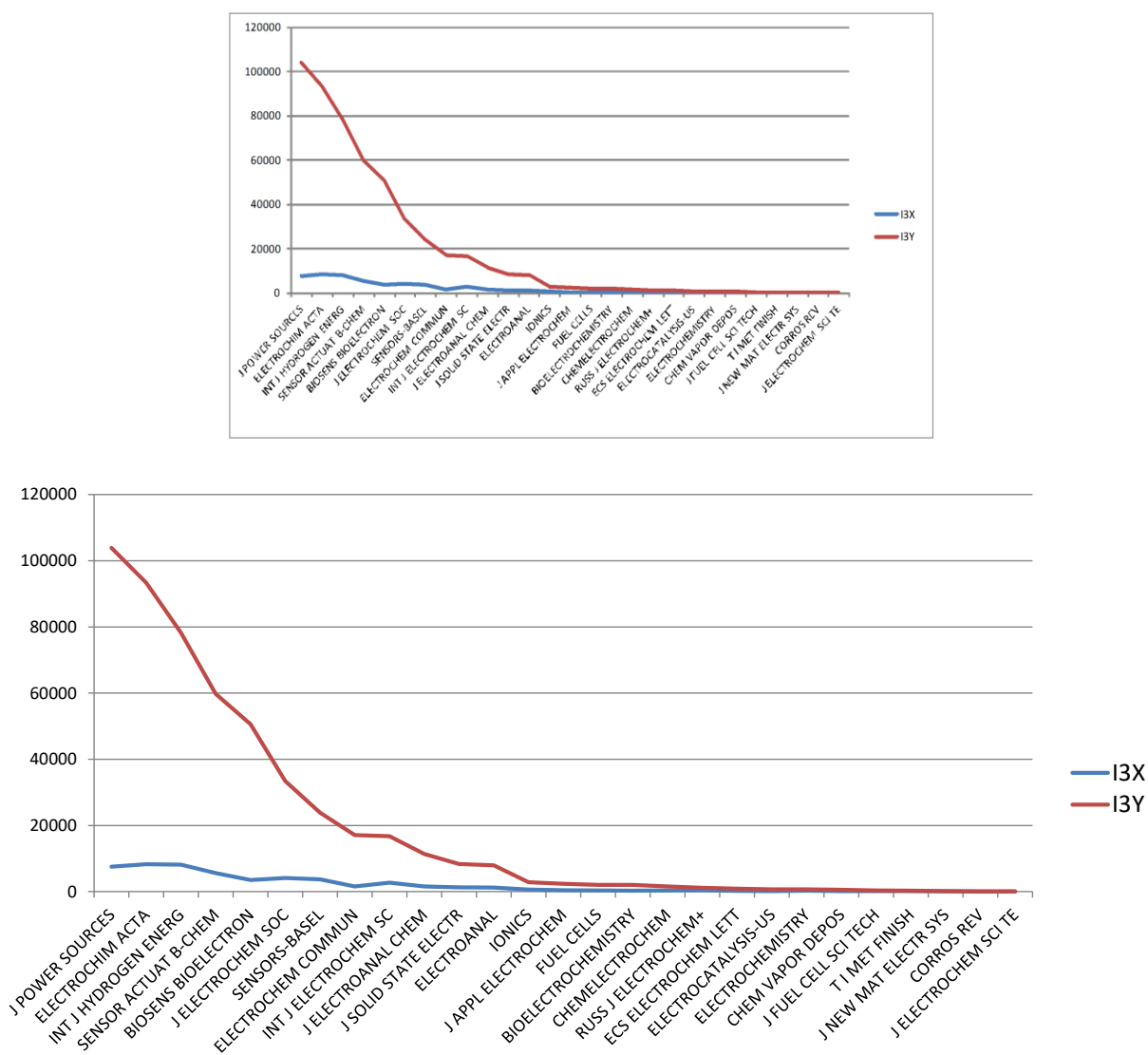


**Fig. 3** The core impact power of 25 famous universities (2011-2015)

Figure 3 shows that Harvard positions the top 1 in core impact power of citations and MIT the top 1 in core impact power of publications, while the Stanford, Berkeley, Cambridge, Oxford follow the tops. Meanwhile, in these top universities, Yale and Michigan have core advantages of publications indicated by obvious peaks.

### 3.3 Group (Massive) level: journals

As all publications and citations are valuable for evaluating in journals, it is reasonable to use I3X and I3Y, which can cover the distribution of publication scores while integrating citation scores of h-core and h-tail, with EC journals as shown as Figure 4 .



**Fig. 4** The I3X and I3Y of 25 EC journals (2011-2015)

In order to understand the relations among all the indicators, Table 1 shows the Spearman correlations between  $h$  and  $\{X_i\}$ ,  $\{Y_i\}$  ( $i=1,2,3$ ),  $IX_3$ ,  $I3Y$  for 25 famous universities and Table 2 provides Spearman correlations between  $JIF$  and  $\{X_i\}$ ,  $\{Y_i\}$  ( $i=1,2,3$ ),  $IX_3$ ,  $I3Y$  for 27 EC journals.

**Table 1** The correlations of multivariate indicators for 25 top-ranked universities (2011-2015)

Correlations		Spearman (Sig.(2-tailed))				
		$h$	$Y_1$	$Y_2$	$Y_3$	$I3Y$
Spearman (Sig.(2-tailed))	$h$	1	.958(.000)*	.838(.000)*	.768(.000)*	.843(.000)*
	$X_1$	.514(.009)*	.678(.000)*	.074(.726)	.824(.000)*	.078(.709)
	$X_2$	.630(.001)*	.440(.028)**	.918(.000)*	.159(.447)	.912(.000)*
	$X_3$	.538(.006)*	.405(.044)**	.775(.000)*	.173(.408)	.778(.000)*
	$I3X$	.671(.000)*	.486(.014)**	.945(.000)*	.188(.369)	.942(.000)*

\*correlation is significant at the 0.01 level (2-tailed); \*\*correlation is significant at the 0.05 level (2-tailed)

**Table 2** The correlations of multivariate indicators for 27 EC journals (2011-2015)

Correlations		Spearman (Sig.(2-tailed))				
		$JIF$	$Y_1$	$Y_2$	$Y_3$	$I3Y$
Spearman (Sig.(2-tailed))	$JIF$	1	.887 (.000)*	.746 (.000)*	.777 (.000)*	.761(.000)*
	$X_1$	.713 (.000)*	.609 (.001)*	.208 (.297)	.593 (.001)*	.233(.242)
	$X_2$	.730 (.000)*	.844(.000)*	.995 (.000)*	.679 (.000)*	.995(.000)*
	$X_3$	-.507 (.007)*	-.275 (.165)	.095(.637)	-.217 (.276)	.068(.735)
	$I3X$	.678(.000)*	.802(.000)*	.988(.000)*	.667(.000)*	.986(.000)*

\*correlation is significant at the 0.01 level (2-tailed)

Table 1 shows that most multivariate indicators (except a few  $X_3$ ,  $Y_3$  and  $I3X$ ) are positively correlated to the  $h$ -index at university level, with Spearman coefficients 0.514, 0.671, 0.843 between  $h$ -index and  $X_1$ ,  $I3X$ ,  $I3Y$  respectively. Table 2 shows similar results: most multivariate indicators (except  $X_3$ ) are positive correlations to  $JIF$  at journal level. Totally,  $\{X_i\}$  ( $i=1,2$ ) and  $\{Y_i\}$  ( $i=1,2,3$ ),  $I3X$  and  $I3Y$  are suitable to be independent indicators.

#### 4. Discussion and Comparison

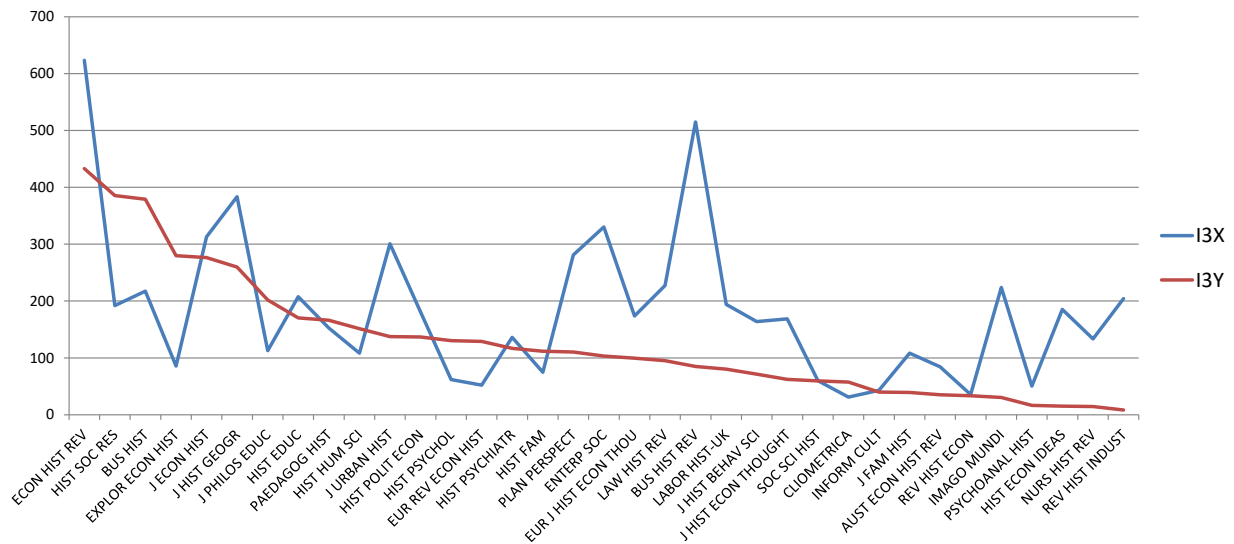
The advantages of  $X_1$  and  $Y_1$  are relative robust like h-index, with non-integral changeability, particularly  $Y_1$  can characterize core impact power of citations. In Table 3, we compare the data of 25 famous universities during the periods of 2009-2013 and 2011-2015, in terms of h-index and  $Y_1$ . One can see the quick development of the Chinese universities compared to world-class universities.

Table 3. The Change of Universities' h-indices and  $Y_1$

2009-2013			2011-2015		
UNIV.	h	$Y_1$	UNIV.	h	$Y_1$
HARVARD	272	4763.45	HARVARD	299	5794.92
MIT	217	4506.3	MIT	241	5374.34
UC BERKELEY	203	3426.45	STANFORD	231	4335.86
STANFORD	202	3242.72	UC BERKELEY	210	3232.96
CAMBRIDGE	190	2822.44	OXFORD	206	2926.63
OXFORD	192	2782.86	CAMBRIDGE	201	2870.43
CHICAGO	164	2387.89	CHICAGO	178	2754.38
MICHIGAN	181	2166.96	TORONTO	200	2654.09
CALTECH	154	2081.41	YALE	183	2464.35
TORONTO	178	2051.62	CALTECH	161	2111.04
YALE	161	1840.59	MICHIGAN	186	2094.91
PRINCETON	133	1559.91	PRINCETON	146	1885.78
TSINGHUA	111	878.081	SYDNEY	153	1608.35
SYDNEY	120	853.671	TSINGHUA	135	1195.78
PEKING	112	799.809	FUDAN	128	1183.3
FUDAN	102	734.071	USTC	120	1098.46
KYOTO	114	714.517	HONG KONG	136	977.568
HONG KONG	116	700.921	PEKING	130	949.031
HUMBOLDT	81	609.58	KYOTO	126	931.677
HAMBURG	82	574.298	ZHEJIANG	126	851.592
USTC	89	552.153	HAMBURG	97	805.082
NANJING	98	487.427	HUMBOLDT	92	789.657
SHANGHAI JIAO TONG	92	459.206	NATL TAIWAN	116	786.014
ZHEJIANG	95	428.322	NANJING	123	759.485

NATL TAIWAN	85	294.895	SHANGHAI JIAO TONG	116	712.446
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There are disciplinary differences, which could affect the applications of the multivariate indicators. For example, comparing the journals of history of the social sciences with the journals of electrochemistry, the relation of I3X and I3Y as well as their correlations to JIF show differences in Figure 5 and Table 4.



**Fig.5** The I3X and I3Y of 35 HSS journals (2011-2015)

**Table 4** The correlations of multivariate indicators for 35 HSS journals (2011-2015)

Correlations		Spearman (Sig.(2-tailed))				
		JIF	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>	I3Y
Spearman (Sig.(2-tailed))	JIF	1	.690 (.000)*	.521 (.001)*	.634 (.000)*	.527(.001)*
	X <sub>1</sub>	.548 (.001)*	.774 (.000)*	.343 (.044)**	.626 (.001)*	.353(.037)**
	X <sub>2</sub>	.470 (.004)*	.347(.041)**	.880 (.000)*	.408 (.015)**	.876(.000)*
	X <sub>3</sub>	.006 (.974)	-.037 (.832)	.084 (.632)	-.172(.323)	.088(.614)
	I3X	.131(.455)	.080(.647)	.348(.041)**	-.041(.813)	.352(.038)**

\*correlation is significant at the 0.01 level (2-tailed); \*\*correlation is significant at the 0.05 level (2-tailed)

Here we see that the correlations in multivariate indicators are much lower in the social sciences. Particularly,  $I3X$  is no longer correlated to  $JIF$ ; it is an independent indicator. Therefore, the multivariate indicators provide richer measurement information than single indicators.

In general, if we want to compare two academic subject or object  $A$  and  $B$ , we may compare all elements of their academic matrices  $V_A$  and  $V_B$ . If all elements in  $V_A$  are better than  $V_B$  (recorded as  $\{V_A\} \succ \{V_B\}$ , not always  $A > B$ ; for  $X3$ , smaller value is better), we can say  $A$  is better than  $B$ . More generally, academic tensor  $T$  is suggested to be a generalized measure including matrix. We can compare all elements of their academic tensors  $T_A$  and  $T_B$ . If all elements in  $T_A$  are better than  $T_B$  (recorded as  $\{T_A\} \succ \{T_B\}$ ), we can say  $A$  is better than  $B$ .

## 5. Conclusions

The multivariate indicators, including publication vector  $\mathbf{X} = (X_1, X_2, X_3)$  and citation vector  $\mathbf{Y} = (Y_1, Y_2, Y_3)$ , publication score  $I3X = X_1 + X_2 + X_3$  and citation score  $I3Y = Y_1 + Y_2 + Y_3$ , as well as their elements and integrated indices, provides a methodological framework for extensive academic measurement. Most of them are positively correlated to the  $h$ -index and  $JIF$ , with relative independence (Spearman coefficients 0.5~0.8), so that they can be considered as independent indicators, which provide multidimensional views for academic evaluation.

$I3X$  and  $I3Y$  combine the advantages of the  $h$ -index and  $I3$ : (i) the publications and not only the citations are appreciated; (ii) the indicators are non-parametric; (iii) the results are easy to obtain from WoS or Scopus data; (iv) the results can be plotted ( $XY$ ).

It is expected to develop further studies in the future.

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## Appendix

Table A1. Scholars' data

Indicator	P	h=P <sub>c</sub>	P <sub>z</sub>	C	C <sub>h</sub>	X1	X2	X3	Y1	Y2	Y3
Leydesdorff L	145	35	15	3673	2404	8.44828	62.2414	1.551724	408.5557	438.4321	378.4484
Ye FY	27	8	4	193	138	2.37037	8.33333	0.592593	21.2228	15.67358	28.37306

Table A2. Publication and citation vectors of 25 famous universities ranked by h-index based on WoS data from 2009 to 2013.

University (ISI Abbreviated Name)	Univ h-index	Publication Vector			Citation Vector		
		X1	X2	X3	Y1	Y2	Y3
HARVARD UNIV	272	1.079165	20582.44	2591.597	3426.448	334689.3	4480.493
MIT	217	1.392601	11333.85	522.5067	2081.409	175299.2	3082.672
STANFORD UNIV	203	0.830076	19064.02	4838.446	2822.444	320858.5	3592.503
UNIV CALIF BERKELEY	202	0.840999	10397.27	5768.135	2387.891	175558.4	6812.783
UNIV OXFORD	192	0.807298	44786.92	8136.022	4763.454	910285.6	2386.228
UNIV CAMBRIDGE	190	0.715319	4545.997	822.7618	574.2979	48509.93	1322.582
UNIV TORONTO	181	0.766562	4067.064	776.5024	609.5801	45841.01	725.9668
UNIV MICHIGAN	178	0.397346	16171.09	2814.811	714.5166	188539.5	637.3312
YALE UNIV	164	0.605866	22933.74	6451.1	2166.962	355131.9	3756.614
UNIV CHICAGO	161	1.52057	18113.27	1612.713	4506.299	310350.5	5967.65
CALTECH	154	0.787154	19806.28	5592.797	2782.855	317658.6	6796.675
UNIV SYDNEY	133	1.135803	8201.824	1099.995	1559.91	112274.4	5373.22
PRINCETON UNIV	120	0.922583	20599.48	4332.119	3242.723	373369.8	2358.782
UNIV HONG KONG	116	0.418131	14795.44	4006.171	853.6706	178022.5	1739.573
TSINGHUA UNIV	114	0.552641	24793.15	6599.81	2051.624	364049	2585.307
PEKING UNIV	112	0.725977	15521.04	4035.065	1840.585	269402.6	1784.917
FUDAN UNIV	111	0.481684	12800.7	2125.225	878.081	127291.7	863.3201
KYOTO UNIV	102	0.442985	14005.98	2426.956	799.8086	152382.1	620.4136
ZHEJIANG UNIV	98	0.485374	10470.28	1882.337	734.0713	113237.5	416.0963
NANJING UNIV	95	0.342165	18748.04	3696.568	700.9207	210327.2	536.6424
UNIV SCI & TECHNOL CHINA	92	0.300191	15794.25	2771.264	487.4272	154049.6	417.7406
SHANGHAI JIAO TONG UNIV	89	0.302189	13202.65	2694.276	459.2057	120447.5	701.8614
NATL TAIWAN UNIV	85	0.231481	14935.64	2913.472	294.8955	145383.4	495.7767
UNIV HAMBURG	82	0.537929	8464.17	818.6611	552.1529	87193.86	335.2364

HUMBOLDT UNIV	81	0.265184	16452.78	3102.155	428.3223	160340.9	223.6169
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Table A2. Publication and citation vectors of 25 famous universities ranked by h-index based on WoS data from 2011 to 2015.

University (ISI Abbreviated Name)	Univ h- index	Publication Vector			Citation Vector		
		X1	X2	X3	Y1	Y2	Y3
HARVARD UNIV	299	1.094619	24314.06	1913.433	3232.96	396695.9	7903.913
MIT	241	1.450045	12384.46	449.9271	2111.042	198999.9	5201.47
STANFORD UNIV	231	0.831177	23130.99	4552.135	2870.434	394468.6	5247.233
UNIV CALIF BERKELEY	210	0.892381	12471.7	5746.589	2754.382	214970	7670.005
UNIV OXFORD	206	0.867213	52946.5	8107.924	5794.924	1042924	5936.042
UNIV CAMBRIDGE	201	0.898491	5597.244	705.9741	805.0824	59230.25	3583.228
UNIV TORONTO	200	0.891792	4919.386	693.7473	789.6574	55358.94	1429.996
UNIV MICHIGAN	186	0.47295	18008.83	2335.886	931.6765	212361.4	828.0422
YALE UNIV	183	0.579624	26893.7	6328.984	2094.907	430861	2882.996
UNIV CHICAGO	178	1.684826	21638.54	1389.099	5374.339	384660.6	9750.973
CALTECH	161	0.786624	24606.31	5552.343	2926.635	413946.2	7558.184
UNIV SYDNEY	153	1.26542	9514.728	968.4489	1885.783	141185.6	5139.476
PRINCETON UNIV	146	1.045679	24906.8	4496.608	4335.864	459288.5	4464.867
UNIV HONG KONG	136	0.567175	18669.88	4325.257	1608.351	236197.7	3317.497
TSINGHUA UNIV	135	0.613459	29408.69	6901.935	2654.091	439022.9	3884.249
PEKING UNIV	130	0.831096	18475.52	4083.185	2464.349	319306.9	3587.126
FUDAN UNIV	128	0.557271	18821.43	1840.817	1195.784	206680.1	1431.567
KYOTO UNIV	126	0.478971	19432.44	2279.871	949.0312	227797.6	1640.276
ZHEJIANG UNIV	126	0.599949	14670.63	1879.865	1183.301	168309.7	1000.845
NANJING UNIV	123	0.414086	23361.4	3347.527	977.5684	282767.1	814.6924
UNIV SCI & TECHNOL CHINA	120	0.356455	23518.15	2710.625	759.4846	245094.5	693.9933
SHANGHAI JIAO TONG UNIV	116	0.361216	19821	2664.592	712.4462	195774.3	1222.995
NATL TAIWAN UNIV	116	0.401708	17416.95	2541.646	786.0145	174981.4	1129.731
UNIV HAMBURG	97	0.763764	11735.74	789.8526	1098.462	133663.5	1277.13
HUMBOLDT UNIV	92	0.368909	23381.48	2908.594	851.592	238996.7	674.8668

Table A3. Publication and citation vectors of 27 journals ranked by JIF in the field of electrochemistry based on WoS data from 2011 to 2015. The journals are ranked by their Journal Impact Factors (JIF) 2015.

Journal (JCR Abbreviated Title)	JIF	Publication Vector			Citation Vector		
		X1	X2	X3	Y1	Y2	Y3

<i>BIOSENS BIOELECTRON</i>	6.395	1.356003	3534.652	1.471358	413.0753	50069.81	120.5079
<i>J POWER SOURCES</i>	5.314	0.937729	7557.725	16.0322	489.1992	103161.5	219.6785
<i>ELECTROCHEM COMMUN</i>	4.417	0.490168	8274.604	47.46639	213.9841	93083.97	32.97644
<i>ELECTROCHIM ACTA</i>	4.119	0.571882	5554.181	16.47085	186.1962	59566.58	37.42595
<i>SENSOR ACTUAT B-CHEM</i>	3.987	1.701574	1568.095	4.106891	380.931	16589.19	152.7751
<i>CHEMELECTROCHEM</i>	3.27	1.91687	284.3056	3.91198	160.8205	1837.389	39.3888
<i>BIOELECTROCHEMISTRY</i>	3.231	0.700971	334.4175	12.73981	53.0407	1533.366	9.778185
<i>J ELECTROANAL CHEM</i>	2.553	0.347822	8080.94	91.32054	138.5712	78089.58	20.19831
<i>J ELECTROCHEM SOC</i>	2.461	0.598673	4032.372	88.23733	241.2083	33041.08	141.447
<i>INT J HYDROGEN ENERG</i>	2.371	0.627907	1535.078	29.79845	110.939	11231.28	42.6959
<i>ELECTROANAL</i>	2.179	0.544135	1208.825	26.66264	77.01084	7842.054	27.3147
<i>J APPL ELECTROCHEM</i>	2.143	1.184426	159.0533	3.688525	60.43488	603.1614	23.44428
<i>J SOLID STATE ELECTR</i>	2.099	0.521432	1265.181	44.07216	84.75002	8287.328	17.20556
<i>ELECTROCATALYSIS-US</i>	2.074	0.714919	427.5391	20.22009	56.6383	2219.753	93.54062
<i>ECS ELECTROCHEM LETT</i>	1.93	0.389484	612.3165	44.59202	41.52608	2826.369	6.075526
<i>CHEM VAPOR DEPOS</i>	1.656	0.488881	3543.361	200.0911	228.0533	23512.43	112.7164
<i>FUEL CELLS</i>	1.648	0.585938	202.7109	21.09375	34.88973	846.0813	9.596141
<i>IONICS</i>	1.627	0.945378	118.5882	12.71008	52.78936	489.2857	2.50365
<i>SENSORS-BASEL</i>	1.571	0.552901	362.6638	19.53754	38.17309	2013.312	1.937818
<i>INT J ELECTROCHEM SC</i>	1.266	0.232688	2554.359	175.3513	48.13264	16719.86	5.17147
<i>CORROS REV</i>	1.05	0.719101	21.75281	15.38202	16.06275	71.47059	12.29804
<i>ELECTROCHEMISTRY</i>	0.714	0.243243	157.1368	127.7449	17.57288	637.0246	24.20424
<i>J FUEL CELL SCI TECH</i>	0.64	0.220109	97.06793	78.53261	11.67438	349.1975	2.569395
<i>T I MET FINISH</i>	0.57	0.146312	269.3881	143.0907	9.959864	1137.312	2.133333
<i>RUSS J ELECTROCHEM+</i>	0.502	0.264706	65.89542	78.51307	13.75472	273.2096	2.568134
<i>J ELECTROCHEM SCI TE</i>	0.462	0.297619	19.04762	18.10714	5.482456	54.74561	0.877193
<i>J NEW MAT ELECTR SYS</i>	0.4	0.172249	34.56938	66.62201	5.355372	152.3306	0.809917