SIMPLE ARITHMETIC VERSUS INTUITIVE UNDERSTANDING: THE CASE OF THE IMPACT FACTOR

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Abstract: We show that as a consequence of basic properties of elementary arithmetic journal impact factors show a counterintuitive behaviour with respect to adding non-cited articles. Synchronous as well as diachronous journal impact factors are affected. Our findings provide a rationale for not taking uncitable publications into account in impact factor calculations, at least if these items are truly uncitable.

Keywords: synchronous and diachronous impact factors; ranking invariance with respect to non-cited items

INTRODUCTION

In this note we show how simple arithmetic may influence our understanding of the impact factor. Concretely, it is possible that the impact factor of journal J is larger than the impact factor of journal J' and that adding the same number of non-cited articles to both reverses the mutual order. Although completely natural from a mathematical point of view, we consider such behaviour as counterintuitive. Indeed, journal J seems more visible than journal J': how can then adding non-cited items make journal J' more visible than journal J?

JOURNAL IMPACT FACTORS

We recall the definitions of the synchronous and the diachronous journal impact factor. The n-year synchronous impact factor of journal J in year Y is defined as (Rousseau, 1988):

$$IF_n(J,Y) = \frac{\sum_{i=1}^{n} Cit(Y,Y-i)}{\sum_{i=1}^{n} Pub(Y-i)} = = \frac{\frac{1}{n}\sum_{i=1}^{n} Cit(Y,Y-i)}{\frac{1}{n}\sum_{i=1}^{n} Pub(Y-i)} \quad (1)$$

In this formula the number of citations received by journal J (from all members of the pool of sources under consideration) in the year Y, by articles published in the year X, is denoted as $CIT_J(Y,X)$, where for simplicity we have not included the index J in equation (1). Similarly, PUB(Z) denotes the number of articles published by this same journal in the year Z. We made it clear in equation (1) that the standard synchronous journal impact factor is a ratio of averages (RoA). Hence we will denote it as RAIF. When n = 2 one obtains the classical Garfield (1972) journal impact factor. Since a few years also the 5-year journal impact factor is provided in Thomson Reuters' Web of Science. The term 'synchronous' refers to the fact that the citation data used to calculate it are data collected in the same year. We next recall the definition of the diachronous impact factor.

The n-year diachronous impact factor of journal J for the year Y, denoted as $IMP_n(J,Y)$, is defined as

$$IMP_n(J,Y) = \frac{\sum_{i=s}^{s+n-1} Cit(Y+i,Y)}{Pub(Y)} \quad (2)$$

where $s = 0$ or 1, depending on whether one includes the year of publication or not. The term 'diachronous' refers to the fact that the data that are used to calculate this impact factor derive from a number of different years with a starting point somewhere in the past and encompassing subsequent years (Ingwersen *et al.*, 2001).

RANKING INVARIANCE WITH RESPECT TO NON-CITED ITEMS

We consider the following form of invariance. If a performance indicator I is calculated for journals J_1 and J_2 and $I(J_1) < I(J_2)$ then, if we add the same number of publications with zero citations, we require that also for the new situation $I(J_1) < I(J_2)$. We refer to this requirement as ranking invariance with respect to non-cited items. This notion is totally different from the consistency notions introduced by Waltman and van Eck (Waltman & van Eck, 2009; Waltman et al., 2011) or by Marchant (2009) (under the name of independence). Recall that, for good reasons, the notion of consistency as defined by these authors refers to cases where the number of publications (in the denominator) is the same for both journals. We do not require this.

Next we show that impact factors are not ranking invariant with respect to non-cited items. Consider the following example (see Table 1).

Table 1: Data for the calculation of the Garfield impact factor (RoA case) for the year Y.

	J_1	J_2
Pub(Y-1)	10 (+25)	30 (+25)
Pub(Y-2)	10	30
Cit(Y,Y-1)	30	60
Cit(Y,Y-2)	30	60

On the basis of Table 1, the Garfield impact factors of journals J_1 and J_2 are $IF_2(J_1,Y) = 3$ and $IF_2(J_2,Y) = 2$, so that $IF_2(J_1,Y) > IF_2(J_2,Y)$. However, adding 25 non-cited publications, yields the new impact factors: $IF_2(J_1,Y) = 60/45 = 1.33$ and $IF_2(J_2,Y) = 120/85 = 1.41$, so that for the new situation the relation between the impact factors reverses.

We note that for the classical synchronous impact factor IF_2 ranking invariance with respect to non-cited items always holds for the special case that both $IF(J_1) < IF(J_2)$ and the sum of Cit(Y,Y-1) and Cit(Y,Y-2) is smaller for journal J_1 than for journal J_2 (or is equal). Indeed: denoting the 2-year impact factor of journal J_i (i = 1,2) simply by C_i/P_i we have:

$$\frac{C_1}{P_1} < \frac{C_2}{P_2} \quad \text{and} \quad C_1 \le C_2$$

If now Z denotes the added number of publications with no citations we have to show that:

$$\frac{C_1}{P_1+Z} < \frac{C_2}{P_2+Z} \Leftrightarrow$$

$$\Leftrightarrow C_1(P_2+Z) < C_2(P_1+Z) \Leftrightarrow$$

$$\Leftrightarrow C_1P_2 + C_1Z < C_2P_1 + C_2Z$$

This is clearly true since $C_1P_2 < C_2P_1$ and $C_1Z \le C_2Z$. We also note that if

$$\frac{C_1}{P_1} < \frac{C_2}{P_2} \text{ and } P_1 \le P_2 \text{ then } \frac{C_1}{C_2} < \frac{P_1}{P_2} \le 1,$$

$$\text{hence also } C_1 \le C_2 .$$

This implies that also under these conditions ranking invariance with respect to non-cited items holds. Denoting by $IF_Z(J)$ the impact factor of journal J when Z non-cited items are added leads us to the following characterization result.

Proposition. If $IF(J_1) < IF(J_2)$ then rank reversal, i.e. $IF_Z(J_1) > IF_Z(J_2)$ occurs if and only if $C_1 > C_2$ and

$$Z > \frac{D}{C_1 - C_2}$$

where $D = P_1C_2 - P_2C_1$.

Proof. We already know that if $C_1 \le C_2$ then there is no rank reversal. If now $IF(J_1) < IF(J_2)$ this implies that $P_1C_2 > P_2C_1$. Its (positive) difference $P_1C_2 - P_2C_1$ is denoted as D. We have now the following equivalences:

$$IF_Z(J_1) > IF_Z(J_2) \Leftrightarrow (P_1+Z)C_2 < (P_2+Z)C_1 \Leftrightarrow$$

$$\Leftrightarrow P_2C_1 + D + ZC_2 < P_2C_1 + ZC_1 \Leftrightarrow$$

$$\Leftrightarrow D < (C_1-C_2)Z \Leftrightarrow Z > \frac{D}{C_1 - C_2}$$

This proves the proposition.

This result shows the exact requirements to have rank reversal in the case of the classical synchronous impact factor, and hence when it does not occur. We continue our investigations by considering the diachronous impact factor. A simple variation of Table 1 shows that also the diachronous impact does not satisfy this property either, see Table 2.

Table 2: Data for the calculation of the dynamic (=diachronous) impact factor (RoA case) for the year Y.

	J_1	J_2
Pub(Y)	20 (+25)	60 (+25)
Cit(Y,Y)	10	20
Cit(Y,Y+1)	20	40
Cit(Y,Y+2)	30	60

With s = 0, we have $IMP_3(J_1,Y) = 60/20 = 3$ and $IMP_3(J_2,Y) = 120/60 = 2$. Adding 25 non-cited publications yields the new diachronous impact factors: $IMP_3(J_1,Y) = 60/45 = 1.33$ and $IMP_3(J_2,Y) = 120/85 = 1.41$. The rank-order of the two journals in terms of their impact factor is thus reversed by adding an equal number of non-cited items to both.

The characterization provided above also holds for the diachronous impact factor as it too is determined by dividing a number of citations by a number of publications.

AoR VERSUS RoA

We have shown that the standard synchronous impact factor is of the RoA-form and that it does not satisfy ranking invariance with respect to non-cited items. Let us analyze whether perhaps an Average of Ratios (AoR) form of the synchronous impact factor behaves better in this respect. First we define the ARIF as:

$$ARIF_n(J,Y) = \frac{1}{n}\sum_{i=1}^{n}\frac{Cit(Y,Y-i)}{Pub(Y-i)} \qquad (3)$$

However, it turns out that the AoR-form behaves even worse with respect to ranking invariance. Indeed, consider the case of a two-year impact factor ($ARIF_2$) and assume that journals J_1 and J_2 have in each year equal numbers of publications. If $RAIF_2(J_1,Y) > RAIF_2(J_2,Y)$, (or, $IF_2(J_1,Y) > IF_2(J_2,Y)$), this means that journal J_1 received more citations than journal J_2 (in the year Y). Adding the same number of zero-cited publications to both, does not change the total number of citations received, and hence J_1's standard impact factor remains smaller than J_2 (of course both impact factors decrease by increasing the denominators). The same argument holds for the n-year synchronous impact factor (RA-case). This, however, does not hold for ARIF. Consider the example shown in Table 3.

Table 3. Data for the calculation of a two-year synchronous impact factor (AoR case) for the year Y.

	J_1	J_2
Pub(Y-1)	30 (+10)	30 (+10)
Pub(Y-2)	20	20
Cit(Y,Y-1)	10	120
Cit(Y,Y-2)	80	10

Based on the data shown in Table 3, we have: $ARIF_2(J_1,Y) = (0.5).(10/30+80/20) = 2.17$ and $ARIF_2(J_2,Y) = (0.5).(120/30+10/20) = 2.25$ so that $ARIF_2(J_1,Y) < ARIF_2(J_2,Y)$. However, adding 10 publications in the year Y-1 yields the new impact factors: $ARIF_2(J_1,Y) = (0.5).(10/40+80/20) = 2.13$ and $ARIF_2(J_2,Y) = (0.5).(120/40 + 10/20) = 1.75$, so that for the new situation $ARIF_2(J_1,Y) > ARIF_2(J_2,Y)$. ARIF is more sensitive to adding publications with no citations to the denominator than RAIF *because* ARIF is an average (cf. Ahlgren et al., 2003); RAIF, however, is not an average, but a quotient between two summations (Egghe & Rousseau, 1996).

It is easy to find similar examples of violations against the assumption of ranking invariance for any n-synchronous impact factor calculated in the AoR way.

A REMARK CONCERNING THE FRAMEWORK OF IMPACT FACTOR CALCULATIONS: "UNCITABLE" ITEMS

When Garfield introduced the impact factor, he decided to introduce the notion of uncitable items. The idea was that journals should not be 'punished' for publishing obituaries, corrections, editorials and similar types of publications, which usually receive no or few citations. Although this seems reasonable, there are in practice two problems with this notion. One is to decide which publications are uncitable, and the other one is the fact that Garfield also decided to include citations to these "uncitable" articles – when they occur - to the total number of received articles. It has been shown, see e.g. (Moed & van Leeuwen, 1995) that this practice may lead to serious distortions in journal impact.

Assume now that if uncitable items could be defined unambiguously, and that they are really never cited, which way of calculating an impact factor is then better? Taking all publications into account (including the – uncited – "uncitable" ones), or taking only the 'citable' ones (cited or not)? The answer is clearly that the second method should be used, as otherwise it would be possible that journal J obtains a higher impact than journal J' due to uncitable publications.

CONCLUSION

We have shown that, as a consequence of simple arithmetic, not satisfying the requirement of ranking invariance with respect to non-cited items is a normal mathematical property related to taking ratios. For the calculation of synchronous impact factors, the standard RoA approach is to be preferred above the AoR approach, as the RoA approach satisfies ranking invariance with respect to non-cited items for journals with the same number of publications, while the AoR approach may fail even in this case. We characterised when standard impact factors fail to have the property of ranking invariance with respect to non-cited articles. Our findings provide a rationale for not taking uncitable publications into account in impact factor calculations, at least if these items are truly "uncitable", that is, are really never cited. Furthermore, they provide another argument against using averages in the case of highly skewed distributions (Ahlgren et al., 2003; Bornmann & Mutz, 2011; Leydesdorff & Opthof, 2011).

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